Examiners' Report
Principal Examiner Feedback

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Pearson Edexcel International A Level
In Pure Mathematics 3 (WMA13)

Paper: 01 Pure Mathematics 3

General

This series is exceptional due to the coronavirus pandemic that caused exams to be delayed from the summer until the Autumn. It also meant many candidates were less well prepared than they normally would be for the exams. However, there were nevertheless plenty of strong performances on the paper with good progress seen in most questions. There was access at all levels into each of the questions.

Report on individual questions

Question 1

This was generally well done with about two-thirds of the candidates scoring full marks. Where errors occurred, these were mainly writing $\cos 2x$ as $\cos^2 x - 1$ or $1 - \cos^2 x$, or incorrectly stating $4\cos^2 x - 2 = 2\cos x$ as the first step, implying an incorrect identity due to a lack of brackets. Sign errors led a few candidates to obtain $\cos x = \frac{1}{4}$ but usually went on to find angles from this, while some omitted or wrongly calculated the reflex angle.
Answers in radians, or spurious extra angles in the required range were rarely seen.

Question 2

The most popular approach to the first part of this question was to attempt to raise as a power of 10 each side of the given equation, and then split the term on the right hand side into the product of two terms from which the values required could be obtained. There were a minority of candidates who thought that the right-hand side became the sum of two terms rather than a product and so lost the last two marks of the part. Often the direct taking of powers was not seen, however, though correct values would imply the first two marks.

A second approach was to take the log to the base 10 of both sides of the required form and then to compare coefficients with the given expression for $\log_{10} N$ before raising to base 10 to find the required constants. In many cases, candidates did not take notice of the accuracy required for the two unknowns. This was not penalised in this case, but candidates should be reminded to give answers to the accuracy required.

Other attempts were seen using natural logarithms rather than using base 10 but these were rarely carried through to a correct conclusion. Also, there were numerous cases of students attempting to raise to base e, and these scored zero marks for the question as they basis understanding that base 10 relates to powers of 10 was required to be demonstrated.

For part (b), most candidates understood how to proceed with most attempting to substitute their values found in part (a) into the given expression for N. There were also candidates who scored both marks for (b) despite making no progress with (a) as they could substitute into the given equation and solve the resulting log equation. Again, the required accuracy was ignored by many candidates.

Question 3

This question provided a variety of responses. Most candidates managed to make some progress in (a) but few went on to gain full credit.

The majority of candidates used the quotient rule correctly to gain the first two marks in part (a) of the question, giving a correct differentiation off(x). Use of the product rule of less frequent but those employing it were still able to access full marks in this question. A small number of candidates failed to quote the quotient rule formula and then gave an incorrect differentiation so were unable to gain credit for their method and subsequently any accuracy marks available. Candidates should be advised to quote the formulae they use in their method.

Most candidates struggled to deal with simplifying their expression to give a linear numerator, with only about one third of candidates scoring the second M mark (and only a quarter obtaining all four marks). Many progressed to expressions with multiple fractions and square roots of (4x-1) but were unable to manipulate these correctly to give a simplified expression. Of those that did manipulate correctly, a few candidates failed to combine $(4x - 1)(4x - 1)^{\frac{1}{2}}$ into $(4x - 1)^{\frac{3}{2}}$ or equivalent single term. This shows that when an expression contains negative fractional indices, students make mistakes in taking out a common factor and use laws of indices correctly.

In part (b) only around a half of the candidates understood that in order to find the range they had to solve f'(x)=0 and find the turning point. However, there were many cases in which students got all three marks without having a simplified f'(x).
Of those not progressing into this part either there was no attempt at all, or many wrongly assumed that $x = \frac{1}{4}$ was the key point and were unable to make further progress. Candidates who correctly solved f'(x)=0 to give x =2 mostly progressed on to find the value of f(2) though not many went on to gain full credit. A minority gave the final inequality as a strict inequality thus losing the final mark only.

Question 4

This question proved accessible to most candidates with many fully correct solutions,

Part (a) required the candidates to evaluate a composite function.
The simplest approach was to evaluate the function in two steps, and this was the most successful method. The commonest error seen in this method was the incorrect application of the modulus function.
The second approach was to attempt to write down an algebraic expression for the composite function first, and then to proceed to evaluate the expression. Errors in this approach usually centred around errors in bracketing and in the application of the modulus function. Over 80% scored both marks for this part.

In attempts to part (b), it was rare to see the use of the given diagram to locate which branch of the graph the intersection lay. Thus, attempts were made to find two intersections by considering both values of | 2- x | and achieving two values of x. Many candidates did not proceed further to decide which of their solutions was the valid one and so lost the final accuracy mark. In some cases, candidates solved just one of the two equations, usually the wrong one. While 80% were able to score the first mark, only 25% scored both.

It was rare to see solutions involving an initial rearrangement of the required equation or ones involving squaring.

Part (c) required two critical values to be found and a resultant inequality given. There was little evidence of the graph being used to help explain candidates' working with many solutions being just the final inequality. Most chose to give the answer as an inequality although the use of set notation was fairly common. Two thirds were able to obtain one correct end of the interval, but only 40% managed both.

The concepts of translation and scaling were understood by the many candidates though only about 50% manage to score marks in part (d). Common mistakes were giving the scale factor as 7 rather than 1/7 and giving the translation as -4 rather than 4.

Question 5

Part (a) of this question was generally approached well by most candidates with nearly three quarters obtaining full marks. Nearly all candidates were able to proceed to sin2$x$cos$x$ + cos2$x$sin$x$ and of those who did not access the first mark (16%) it was mainly due to complete omission of the question. Most candidates could then use the double-angle formula for sin2x correctly. For cos2x some did not use the preferred identity at first but in most cases, they recovered this later to give everything in terms of sin$x$. Only very few lost the final mark for poor notation having done the rest correctly, but candidates should still be reminded that a proof question requires every line in their working to be mathematically correct with correct notation and bracketing.

Part (b) was less successfully completed, with less than half of the candidates obtaining more than the first two marks. This may be in part due to the fact that integration is a new topic on P3 in this specification. However, it was striking that very many candidates did not look carefully at the result from part (a) and what they were asked to do and make the correct link.
Not much over half the candidature used the identity in part (a) correctly and integrated an expression of the form $A \sin x + B \sin 3x$, though there were also a few who managed to correctly apply the alternative method of the scheme. Candidates should be reminded that when a question uses the word "hence" they should be looking to use their result or the printed result from the previous part of the question. In this instance alternative methods were accepted, but they may not be in future assessments.
Most who used the identity went on to integrate to an expression of the correct form, but some had incorrect coefficients (usually having multiplied, rather than divided, the second term by 3).
Those who did not make progress into this part generally fell into two camps. Some tried to integrate the RHS of the expression given directly, including the sin³x term, often getting this latter term incorrect. Others attempted to just integrate sin³x directly, without applying an identity first, usually achieving $\frac{\sin^4 x}{4}$, though other incorrect expressions were also seen. Also, some tried integrating using substitution with $u = \sin x$, but their attempts were unsuccessful.

Question 6

In part (a), most candidates understood how to proceed to find the x coordinate of the required point. It was disappointing to see the high proportion of candidates who got a correct solution to the equation but either did not notice the requirement to give the answer in the form $\ln k$ or did not appreciate how to proceed from $1 + \ln 3$ to the required form, with 85% scoring the first two marks but only 40% achieving the third.

Part (b) proved much more challenging for candidates with full marks being a rare occurrence, the final A being scored by only 12%. Even the first two marks in this part only had 40% success rate.

A suitable interval was correctly identified by the majority with 1.1335 and 1.345 being most commonly used, but this alone was not sufficient to score a mark. It was rare to see a tighter interval being used although that did gain credit if used properly. Some candidate used an interval that was too wide e.g. 1.13 to 1.14, so could not access the method. However, some candidates saw this as a question about iteration and chose to attempt to rearrange into the form $x = f(x)$ from which they could not proceed. A careful reading of the question must be made, rather than assuming what is being asked.

The most commonly used suitable functions were $\pm(7 - x^2 - 5e^{x-1})$ with other possibilities seen only rarely. However, in some cases candidates failed to state this as the function, and so could not gain the initial B mark, but had correct values implying this value.

A small number of candidates chose to compare the values of the y coordinates for each curve at either side of the root thus ignoring the requirement to state a suitable function. It was rare to see such attempts carried through completely with an acceptable conclusion.

Many otherwise sound solutions lost the final mark because the conclusion that the root was within the stated interval did not make any reference to the continuity of the function being considered. Candidates should be reminded that this is an important part of the explanation of the location of the root.

The method of finding the second iterate in part (c) was well understood with 84% scoring the first mark. However, there were candidates who did not give the value of their iterate to the required number of decimal places as only 74% achieved the correct answer. Only around a third solutions then proceeded to evaluate further iterates to get the value of β, although there was often no intermediate working to be seen with the value of β just being stated.

Question 7

Part (a) was done well with only about 20% of candidates failing to score all three marks. The most common fault was finding $\tan\alpha = \frac{1}{4}$ and so producing an incorrect value for α. A minority of candidates failed to calculate R correctly, usually forgetting to square the 4 and obtaining $\sqrt{5}$. A few candidates gave $\sqrt{17}$ as a decimal and a few gave the angle in degrees, but this was rare.

Part (b) did cause some confusion for many in that they did not realise that for the fraction to be a minimum the denominator had to be a maximum and so $\cos\left(\frac{1}{2}t - 1.326\right)$ needed to be 1. Instead there were several cases where there was an attempt to make the denominator as small as possible and there was a significant number of candidates who thought this smallest value was 3-$\sqrt{17}$ and did not

comment on the ensuing negative answer, which made no sense in context. Some candidates tried to answer the question without using part (a) to simplify the denominator, usually resulting in $\frac{24}{3+0+4} = \frac{24}{7}$ as a result, and $\frac{24}{3}=8$ was also a quite common answer.

Even when candidates realised that $\cos\left(\frac{1}{2}t - 1.326\right) = 1$ was needed, and obtained $\frac{24}{3+\sqrt{17}}$ many did not correctly interpret this to an answer of sufficient accuracy in cm. Most often it was taken to be 3cm to the nearest cm, as candidates assumed the units were cm. Only about a third of the candidates achieved the correct answer for this part.
Very few tried the differentiation approach for part (b) (and did not proceed very far).

Part (c) proved more accessible and most realised the need to use the result from part (a). There was a success rate of about 50% of candidates scoring all four marks. The most common mistakes were forgetting to take arccos once the $-0.6/\sqrt{17}$ was established and dividing by 2 instead of multiplying by 2 in the final stage.

Question 8

Part (i) of this question required the candidates to find the derivative of the product of two functions and then use the derivative to find the x coordinate of the stationary point of the curve.
In finding the derivative in part (a), most candidates chose to use the product rule. The form of the product rule was usually correct, and a correct derivative was often found by over two thirds of candidates. Errors included the omission of a factor of 3 in the differentiation of e^{3x} and of 2 in the differentiation of sec $2x$. Also, there were instances of candidates changing the argument of any differentiated trigonometric functions from $2x$ to x.
On seeing the function sec $2x$, some candidates chose to rewrite this as the reciprocal of $\cos 2x$ and then proceed to use the quotient rule. Those candidates choosing this method could usually achieve a correct form of their derivative.
Part (b) required the solution of a trigonometric equation. It was very rare to see solutions using degrees. However, it was more common to see the answer being rounded too severely and thus losing an accuracy mark if a more accurate value was not seen, with 60% successfully achieving the methods but only 30% the correct answer. Candidates should be reminded that the instructions on the front of the examination paper require inexact answers to be given to three significant figures unless otherwise stated.
In solving the trigonometric equation, with its roots in the second and fourth quadrants, candidates who chose to work with a base angle in the first quadrant and from that generate the required solutions often made errors.

Candidates found part (ii) of this question much more challenging, with only about one third gaining full marks.
Two main methods of solution were attempted. The first was that of the main scheme, differentiating with respect to y first to find $\frac{dx}{dy}$ before reciprocating each side. However, it was not uncommon to see the cos$y$ term missing in such methods. Also, in those who did obtain the derivative correctly, many were unable to write cos$y$ in terms of e^x.
The second method required was to first proceed to $e^x = \sin y$ and then use implicit differentiation as its first step. Following this, there was a requirement to achieve an expression for $\frac{dy}{dx}$ and then use an appropriate trigonometric identity to transform $\frac{dy}{dx}$ into a function of x only.
Again, it was surprising to see the large proportion of candidates using this method who reached an answer e^x / cos y and proceeded no further to replace cos y in terms of x.

A further method first required the candidate to rewrite the given function into the form $y = \arcsin(e^x)$ and then use the standard form for the derivative of the arcsin function together with the chain rule to achieve the required result. However, this uses a result that is beyond the specification so should not be encouraged.

Overall, about two thirds of the candidates achieved the first two marks, but little more than a third scored the latter two.

Question 9

There were many good attempts at this question and although there were also some blank responses most candidates did make some attempt at it. It is unfortunate that some appeared to run out of time to complete all three parts as this question was worth almost 19% of the total score for the paper so could have a significant impact on a candidate's results.

Part (a) was generally successfully attempted with over 70% scoring full marks. The first M and A marks were usually awarded, and most candidates chose to do long division using the quadratic denominator. However, some candidates did not continue to factorise the denominator and cancel $(x+3)$ and hence not proving that Q is 5, they just stated it instead, losing the second last two marks of part (a). Candidates need to be aware that when a question asks them to "show that", then it is important to justify the answer.

Partial fractions were seen as an alternative method for dealing with the remainder.

The alternative method in the scheme of multiplying though and comparing coefficients or substituting values was also popular. A few different approaches were seen here and almost all successful, but again there were candidates who failed to show sufficient working to justify the value of Q.

Others tried to use a standard 'partial fractions' method for the whole expression, which was incorrect as its improper nature had not been taken into account, so they could not obtain full marks.

Division separately by the factors $(x+3)$, $(x-4)$ was not usually successful, although there was some intelligent partitioning of the numerator that achieved the correct result.

Part (b) was also well done by the vast majority of candidates, with the methods scored by over 70% though only 50% scored full marks. Most achieved a correct form for the derivative, though occasionally there was a sign error on the $(x-4)^{-1}$ term. But not all candidates managed to differentiate $(x-4)^{-1}$ to a correct form, with some attempting the quotient rule and erroneously differentiating 5 to 5, resulting in an incorrect form for the answer. Others thought it was $\ln(x-4)$ (but usually gave the integral of $(x-4)^{-1}$ as $\ln(x-4)$ in part (c) too!)

A small number of candidates started from the original expression for g(x) and used the quotient rule, usually successfully.

Most demonstrated a correct process to find the equation of the tangent, evaluating g'(2) for the gradient and using to form the equation. Use of $y - y_1 = m(x - x_1)$ and of $y = mx + c$ were equally popular. After forming a correct formula for g'(x), there were sometimes errors when substituting into the $-5(x-4)^{-2}$ version and the final mark was lost. Use of $y = mx + c$ sometimes resulted in the omission of a final equation.

The two main reasons for loss of the final method mark were for either using g(0) instead of g(2) for the y value, or attempting the equation of the normal instead of the tangent.

(c) Most candidates, if they got this far, generally gained the first 3 marks (roughly 50%), but many did not offer attempts. Two thirds accessed at least the first method mark, and well over half progressed to through the first three marks. Sorting out the logarithms caused some issues towards the end - many were unaware they needed the log of the modulus of the linear term in x. Some candidates rewrote the original fraction as $...-\dfrac{5}{4-x}$ then integrating this to $-...-5\ln(4-x)$ most likely thinking ahead

to the limits of integration, that this will eliminate a negative log, but ended up with a sign error as a result. Occasionally the whole function was combined and an incorrect method ensued, but this was rare.

For evaluating the definite integral, some candidates assumed the bottom limit of 0 would give 0, and showed no substitution, thus losing the final three marks. But the majority of candidates substituted the limits in correctly and in the right order. However many did not have the modulus and left the values as $5\ln(-2)$ and $5\ln(-4)$ not realising these are undefined (though recovery through correct log work was permitted). Some of these then tried to bring out the minus sign, to $-\ln(2)$ etc. and so their answer went wrong, and they ended up with a final result with changed signs.

The combining of ln terms, even when correct, was completed in a variety of ways, such as

- $5\ln4$ to $10\ln2$ and then subtracting terms
- $\ln(\pm2) - \ln(\pm4)$ to $\ln(1/2)$ then to $\ln1 - \ln2 = -\ln2$
- $\ln2^5 - \ln4^5 = \ln1/32$ to $-\ln32 = -\ln2^5 = -5\ln2$

Answers of $20/3 + 5\ln(1/2)$ were quite common and candidates either did not know how to proceed to form required or thought this was an acceptable alternative.

Only one third of candidates achieved a fully correct result.